Section 4.4: Identity and Inverse

Identity Matrix

The *n* x *n* identity matrix is the matrix that has 1's on the main diagonal and 0's elsewhere

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad I = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Matrix

Two *n* x *n* matrices are inverses of each other if their product (in both orders) is the *n* x *n* identity matrix

$$A = \begin{bmatrix} -3 & -7 \\ 2 & 5 \end{bmatrix} \quad B = \begin{bmatrix} -5 & -7 \\ 2 & 3 \end{bmatrix}$$

$$A \times B = \begin{bmatrix} -3 & -7 \\ 2 & 5 \end{bmatrix} \times \begin{bmatrix} -5 & -7 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$B \times A = \begin{bmatrix} -5 & -7 \\ 2 & 3 \end{bmatrix} \times \begin{bmatrix} -3 & -7 \\ 2 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

yes, A and B are inverses.

$A = B^{-1}$

$B = A^{-1}$

Finding Inverse Matrix

For a 2 x 2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^{-1} = \frac{1}{|A|}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix} = \frac{1}{ad - cb}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$|A| \neq 0$

opposite

$$A = \begin{bmatrix} -3 & -7 \\ 2 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{1}{(-3 \bullet 5) - (2 \bullet -7)}\begin{bmatrix} 5 & 7 \\ -2 & -3 \end{bmatrix} = (-1)\begin{bmatrix} 5 & 7 \\ -2 & -3 \end{bmatrix} = \begin{bmatrix} -5 & -7 \\ 2 & 3 \end{bmatrix} = A^{-1}$$

$$A = \begin{bmatrix} 2 & 3 \\ 3 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{1}{(2 \bullet 4) - (3 \bullet 3)} \begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix} = \frac{1}{-1} \begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix}$$

$$= -1 \begin{bmatrix} 4 & -3 \\ -3 & 2 \end{bmatrix} = \begin{bmatrix} -4 & 3 \\ 3 & -2 \end{bmatrix} = A^{-1}$$

Solve for the 2 x 2 Matrix X $\begin{bmatrix} 3 & 2 \\ 4 & 3 \end{bmatrix} X = \begin{bmatrix} 12 & 2 \\ 17 & 3 \end{bmatrix}$

Find the inverse of the first matrix

$$\frac{1}{(3 \bullet 3) - (4 \bullet 2)} \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix}$$

Multiply the Inverse to the second matrix on the ***left***

$$X = \begin{bmatrix} 3 & -2 \\ -4 & 3 \end{bmatrix} \times \begin{bmatrix} 12 & 2 \\ 17 & 3 \end{bmatrix}$$

$$X = \begin{bmatrix} 2 & 0 \\ 3 & 1 \end{bmatrix}$$

Solve for the 2 x 2 Matrix X $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} X = \begin{bmatrix} 12 & 2 \\ 16 & -5 \end{bmatrix}$

Find the inverse of the first matrix

$$\frac{1}{(2 \bullet 3) - (1 \bullet 2)} \begin{bmatrix} 3 & -2 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 3/4 & -1/2 \\ -1/4 & 1/2 \end{bmatrix}$$

Multiply the Inverse to the second matrix on the ***left***

$$X = \begin{bmatrix} 3/4 & -1/2 \\ -1/4 & 1/2 \end{bmatrix} \times \begin{bmatrix} 12 & 2 \\ 16 & -5 \end{bmatrix}$$

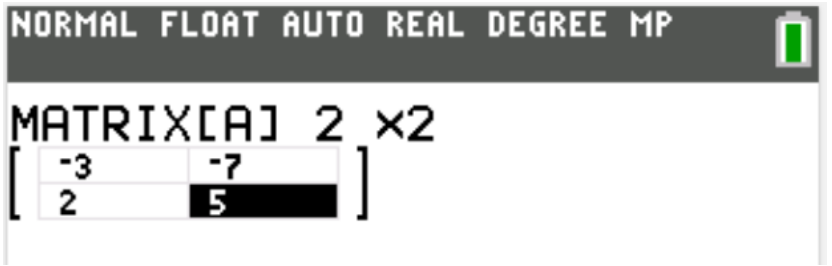
$$X = \begin{bmatrix} 1 & 4 \\ 5 & -3 \end{bmatrix}$$

Practice: p. 227 #13,15,17,25,27

Using the calculator

Find the inverse $A = \begin{bmatrix} -3 & -7 \\ 2 & 5 \end{bmatrix}$

Enter matrix into calculator



In home screen bring up matrix ID and press x^{-1}

Solve for the 2 x 2 Matrix X $\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix} X = \begin{bmatrix} 12 & 2 \\ 16 & -5 \end{bmatrix}$

Enter first matrix into [A] and second matrix into [B]

Multiply the Inverse to the second matrix on the ***left***

$$X = A^{-1} \bullet B$$

$[A]^{-1} * [B]$

$$\begin{bmatrix} 1 & 4 \\ 5 & -3 \end{bmatrix}$$

$$X = \begin{bmatrix} 1 & 4 \\ 5 & -3 \end{bmatrix}$$